NUMERICAL SOLUTION OF THE PROPAGATION OF CYLINDRICAL SHOCK WAVES IN A MAGNETIC FIELD

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Laboratory research on thermonuclear synthesis [1], certain astrophysical processes [2], and active experiments in space [3] involve the propagation in a magnetic field of strong shock waves, which ionize the surrounding medium. It is therefore of interest to study the coupling of ionizing shock waves and the magnetic field.

The propagation of a cylindrical gas cloud and a strong cylindrical shock wave was considered in [4, 5]. The collisionless expansion of an ionized cloud into a homogeneous plasma was studied in [4]. The propagation of a cylindrical shock wave was solved in [5] with the effects of a magnetic field and radiation taken into account. A numerical solution for the explosion of a spherical charge in a magnetic field was considered in [6] in the approximation of small Reynolds numbers (and without the deformation of the initial magnetic field taken into account). The solutions of several problems involving explosions with the effect of a magnetic field taken into account were given in [7] for the case of a point explosion. In [8] the propagation of ionizing shock waves in a uniform magnetic field was studied in the approximation that all of the gas is near the surface of the front.

In the present paper we solve the system of magnetohydrodynamical equations numerically and analyze the effect of a magnetic field on the propagation of cylindrical shock waves in a completely ionized gas and in a weakly ionized gas modeling the atmosphere of the earth.

It was shown in [9] that the loss of energy due to ionization of the air from the expansion of the heated plasma cloud is relatively small, and therefore the energy losses due to radiation and ionization of the surrounding gas are not taken into account in the treatment given here.

We consider the gasdynamical expansion of a cylindrical plasma cloud into a rarefied medium. The magnetic field vector is along the axis of the cylinder. Then the problem becomes one-dimensional and is specified by the system of magnetohydrodynamical equations in terms of the cylindrical coordinates r, φ, z

$$\frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\mathbf{v})=0; \tag{1}$$

$$\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}=-\frac{\nabla p}{\rho}+\mathbf{f},\ \mathbf{f}=\frac{1}{c}[\mathbf{j}\times\mathbf{H}]; \tag{2}$$

$$\frac{\partial e}{\partial t}+(\mathbf{v}\nabla)e=-\frac{p}{\rho}\operatorname{div}\mathbf{v}+\frac{1}{\rho}(\mathbf{j}\cdot\mathbf{E}); \tag{3}$$

$$\frac{\partial\mathbf{H}}{\partial t}=\operatorname{rot}[\mathbf{v}\times\mathbf{H}]-c\operatorname{rot}\mathbf{E}; \tag{4}$$

$$\mathbf{j}=\frac{c}{4\pi}\operatorname{rot}\mathbf{H}, \tag{5}$$

where ρ is the density, p is the pressure, $\mathbf{v}$ = (v, 0, 0) is the velocity, e is the internal energy, $\mathbf{E}$ = (0, E, 0) is the electric field, $\mathbf{H}$ = (0, 0, H) is the magnetic field, $\mathbf{f}$ = (f, 0, 0) is the pondermotive force, $\mathbf{j}$ = (0, j, 0) is the current density, c is the speed of light, and t is the time.

The gas is assumed to be inviscid and non-heat-conducting. It is convenient to consider the flow of the gas governed by the system (1) through (5) in terms of Lagrangian variables ($dm = \rho r dr$):

$$\frac{1}{\rho}=r\frac{\partial r}{\partial m}; \tag{6}$$

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$$\frac{\partial v}{\partial t}+r\frac{\partial}{\partial m}\left(p+\frac{H^2}{8\pi}\right)=0,\ v=\frac{\partial r}{\partial t}; \tag{7}$$

$$\frac{\partial e}{\partial t}+p\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=\frac{1}{\rho}(jE); \tag{8}$$

$$\frac{\partial}{\partial t}\left(\frac{H}{\rho}\right)=-c\frac{\partial}{\partial m}(rE); \tag{9}$$

$$j=-\frac{c}{4\pi}\rho r\frac{\partial H}{\partial m}. \tag{10}$$

In order to close the system of equations (6) through (10) we use the equation of state of an ideal gas

$$p=\rho RT,\ e=p/[\rho(\gamma-1)] \tag{11}$$

and the generalized Ohm's law

$$j=\sigma E. \tag{12}$$

Here R is the universal gas constant, γ is the adiabatic index, T is the temperature, and σ is the conductivity.

In view of the symmetry of the problem, we seek the solution in the region $r > 0$ with the boundary conditions $r = 0$, $v = 0$, $E = 0$ and at infinity with $v = 0$, $H = H_0$, $\rho = \rho_0$, $p = p_0$, where H_0 is the external magnetic field, and ρ_0 and p_0 are the initial density and pressure of the surrounding gas.

At the initial time the plasma is inside a cylindrical surface of radius r_0 with the parameters ρ_1, p_1, T_1, v, where $p_1 > p_0$ and $T_1 > T_0$. The initial magnetic field in the cloud is H_1.

Because of excess pressure the plasma cloud begins to expand (assuming that $p_1 + H_1^2/8\pi > p_0 + H_0^2/8\pi$) and form a shock wave, which heats the surrounding gas and also ionizes it when the temperature reaches a certain critical temperature T_*. Because of the motion of the electrically conducting gas transverse to the magnetic lines of force, an inductive current is created which leads to a braking of the expanding cloud by the ponderomotive forces, and to a change in the initial magnetic field.

We transform the MHD equations to dimensionless form. As the basic units we use r_0, ρ_0, p_0, R. The units of the remaining quantities can be expressed in terms of the basic units as follows:

$$v_0=(\gamma p_0/\rho_0)^{1/2},\ t_0=r_0/v_0,\ \varepsilon_0=p_0/\rho_0,$$
$$T_0=p_0/R\rho_0,\ H_0=p_0^{1/2},\ E_0=v_0H_0/c,$$
$$j_0=cH_0/r_0,\ \sigma_0=c^2/(v_0r_0).$$

In the material below, all quantities (except for the initial basic parameters) are expressed in dimensionless units.

The problem (6) through (12) was solved numerically by the finite-difference method. The gasdynamical part of the difference equations was constructed according to [10] with the introduction of linear and quadratic viscosities. The functions r and E are defined at integral time points and at the centers of the mass intervals; the functions p, ρ, T, and e at integral time points and "half-integer" points of the mass intervals, and v is defined at "half-integer"time points and "half-integers" points of the mass intervals. At each time step we first calculate the gasdynamical functions explicitly, then by the method of [11] we solve the implicit electromagnetic field equations and find E and H. The time step is chosen from the Courant condition, in which the speed of the fast-mode magnetic waves plays the role of the speed of sound, and the condition involving the artificial viscosity [10].

The expansion of a plasma cloud into a completely ionized background medium was solved using this numerical method. The conductivity in this case takes the form [12]

$$\sigma=\sigma_*T^{3/2}.$$

Air was taken as the surrounding gas at $p_0 = 0.18\cdot10^{-2}$ dyn/cm², $\rho_0 = 0.43\cdot10^{-12}$ g/cm³. The initial radius of the cylindrical plasma cloud was $r_0 = 0.5\cdot10^5$ cm and $H_0 = 0.5$ Oe.

Figure 1 shows the time dependence of the radius of the cloud and the boundary of the shock wave front for $p_1 = 10^2$, $\rho_1 = 1$, $v_1 = 0$, $H_1 = 0$. Curves 1 and 4 characterize the

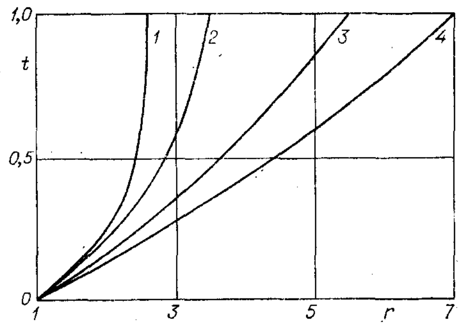

Fig. 1

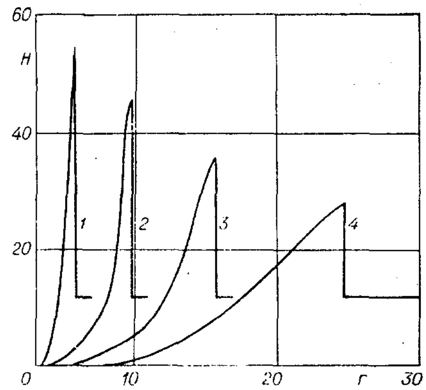

Fig. 2

position of the contact boundary and the shock wave front when the effect of the magnetic field is taken into account in the expansion of the plasma cloud and the propagation of the shock wave. Curves 2 and 3 show the gasdynamical calculations without the effect of the magnetic field taken into account. As seen from Fig. 1, the magnetic field slows the expansion of the plasma cloud, however the shock wave front propagates more rapidly. In the limiting case of small perturbations this corresponds to the fact that the speed of propagation of fast-mode MHD waves is $(a_s^2 + H^2/(4\pi\rho)^{1/2})^{1/2}$, where a_s is the speed of sound in the gas. The motion of the contact boundary is slower when the magnetic field is taken into account because of the effect of the ponderomotive forces.

The magnetic field distributions for the times t = 0.13, 0.34, 0.80, 1.72 (curves 1 through 4) are shown in Fig. 2 for $p_1 = 10^4$, $\rho_1 = 1$, $v_1 = 0$, $H_1 = 0$. We see that the field is expelled from the volume occupied by the plasma. Hence because of the high conductivity of the plasma a condition close to the frozen condition is realized, and the peak in the density distribution corresponds to the peak in the magnetic field distribution.

Calculations done for zero initial magnetic field in the plasma ($H_1 = 0$) and for an initial field $H_1 = H_0$ show that there is no significant difference in the behavior of the solutions for these cases.

Figure 3a shows the dependence of the gasdynamical pressure p, the magnetic pressure $H^2/8\pi$, and the total pressure $p + H^2/8\pi$ for t = 1.9 ($p_1 = 10^2$). A cusp of the gasdynamical pressure on the shock-wave front is characteristic for shock-wave propagation without the effect of the magnetic field taken into account. From our calculations it follows that there is a jump in the pressure of the plasma (curve 1) on the shock-wave front for the case of propagation in a magnetic field, and the cusp corresponds only to the total pressure (curve 3). Curve 2 of Fig. 3 shows the magnetic pressure distribution. These gasdynamical pressure and magnetic pressure distributions are established in the later stages of the propagation of the shock wave.

Figure 3b shows the dependence of the gas pressure p with (solid curve) and without (dashed curve) the inclusion of the magnetic field for t = 0.13, 0.25, 0.34 (curves 1 through 3) with $p_1 = 10^4$. It is seen that there is a difference in the qualitative behavior of the gas pressure when the effect of the magnetic field is taken into account.

We also considered the expansion of a plasma cloud in the earth's atmosphere with the parameters $p_0 = 0.18 \cdot 10^{-2}$ dyn/cm², $\rho_0 = 0.43 \cdot 10^{-12}$ g/cm³, $H_0 = 0.5$ Oe, and $r_0 = 10^5$ cm. Shock

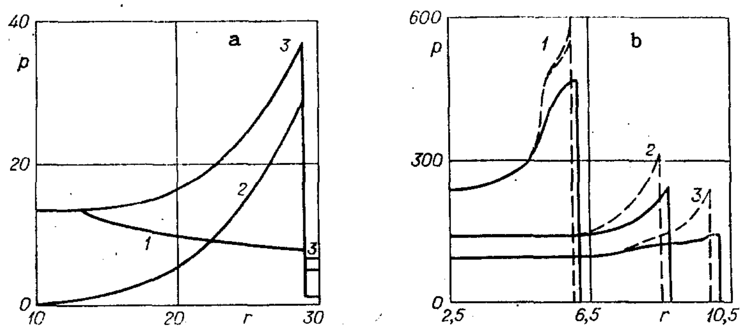

Fig. 3

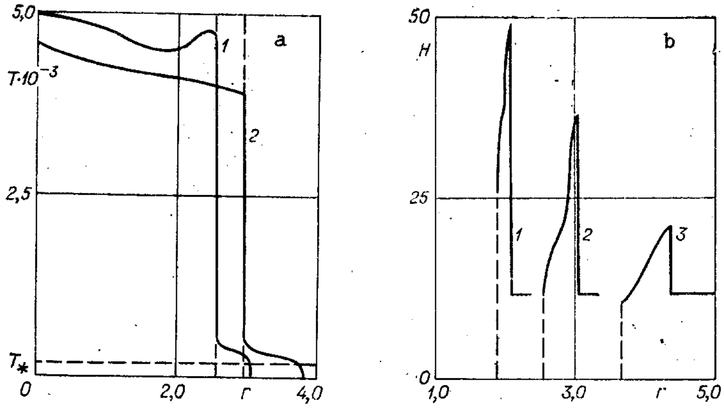

Fig. 4

waves propagating with respect to the surrounding air heat and ionize the gas. The inclusion of the ionization leads to the necessity of taking into account the structure of the shock wave because the usual conservation laws do not in general give a complete set of boundary conditions on the shock wave of ionization. In [13] a numerical model of the flow was presented which allows one to avoid the direct calculation of the ionization front. It is assumed that ionization occurs when the gas reaches a certain critical temperature T_* (in the calculations $T_* \sim 30$ eV) and therefore the ionization front is the surface $T = T_*$. The background gas has the constant conductivity σ_1 induced by photoionization, for example. We will assume that behind the ionization front the gas is completely ionized and the conductivity of the plasma has the form $\sigma = \sigma_2 T^{3/2}$.

Therefore the inclusion of ionization leads to the following temperature dependence of the conductivity:

$$\sigma = \begin{cases} \sigma_1, & T < T_*, \\ \sigma_2 T^{3/2}, & T > T_*. \end{cases} \tag{13}$$

As before we will apply Ohm's law (12), whose applicability for the heights of 90 km and above considered here follows from the data of [14]. In addition, ahead of the ionizing shock wave front there is always a region of initial ionization, which causes a rise in the concentration of charged particles. Therefore the Hall parameter $\chi = \omega^e \tau^e \ll 1$ and this means that the region of applicability of Ohm's law in the form (12) can be expanded to include heights below 90 km. The coefficients in (13) were taken to be $\sigma_1 = 10^{-6}$, $\sigma_2 = 10^2$.

Figure 4a, b show the temperature distributions for t = 0.08; 0.12 (curves 1 and 2) and magnetic field distributions for t = 0.04; 0.08; 0.2 (curves 1 through 3) at $p_1 = 10^4$. The vertical dashed lines denote the boundary of the plasma cloud; the horizontal dashed line correspond to $T = T_*$. The step in the temperature distribution in front of the contact boundary appears because of the heating of the surrounding gas by the shock wave. The temperature behind the shock-wave front is above T_* for the times considered here, and hence the shock wave ionizes the surrounding gas. Because of the high conductivity of the plasma the magnetic field behaves in the flow region in a way similar to the case of a totally ionized medium as considered above. However the magnetic field maxima are smaller and they fall off more rapidly than in the case of a completely ionized surrounding medium.

Figure 5 illustrates the behavior of the current density behind the shock-wave front (curves 1 through 3 correspond to t = 0.04; 0.05; 0.08). The positive and negative maxima appear because of the discontinuity in the magnetic field on the contact boundary, and the discontinuity on the shock-wave front.

The calculations of the flow for $p_1 = 10^2$ show that the shock wave heats the surrounding gas to temperatures below T_* and ionization does not occur. Because of the small conductivity of the surrounding gas ($Re_m = 4\pi\sigma_1 \ll 1$) the induced magnetic field is negligibly small in comparison with H_0. But the field is expelled from the plasma cloud and here $Re_m \gg 1$. Figure 6 shows the gasdynamical pressure distribution p for t = 1.6; 2.4 and $H_0 = 11.7$ (curves 1 and 2) and for t = 2.4 and $H_0 = 0$ (curve 3). The interaction of the external magnetic field on the contact surface with the plasma leads to a strong braking of the plasma ($\beta = 8\pi p/H_0^2 \ll 1$) at the times considered here. The high-pressure region inside the cloud is maintained by the external magnetic pressure. Because of the strong braking of the cloud by

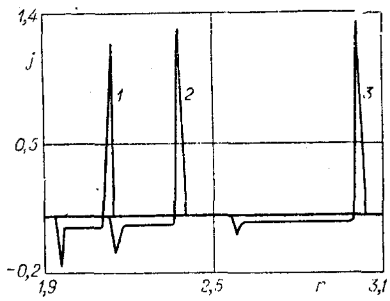

Fig. 5

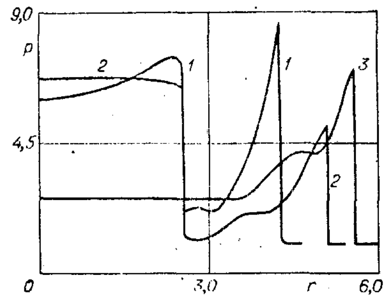

Fig. 6

the magnetic field, the shock wave traveling in the surrounding gas is much weaker and has a front velocity smaller than in the case when there is no external magnetic field.

Calculations for different values of the coefficients σ_1 and σ_2 in (13) show the variation of these coefficients ($\sigma_1 = 10^{-4}$, 10^{-8}, $\sigma_2 = 10^4$) does not lead to a significant difference in the results.

Hence it follows from our calculations that the magnetic field changes the qualitative and quantitative behavior of the physical characteristics for the propagation of shock waves in a completely ionized and a weakly ionized surrounding medium. There is also a slowing of the contact boundary during the expansion of the plasma cloud, and because of the high conductivity of the plasma (where the conditions close to the frozen condition are realized) the field is expelled from the cloud. There are differences in the qualitative behavior of the gasdynamical pressure when the magnetic field is taken into account. The behavior of the magnetic field and the velocity of the shock-wave front depend on the degree of ionization of the surrounding gas.

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